# Evaluating a Polynomial Using the Remainder Theorem

The **Remainder Theorem** states that if a polynomial is divided by , then the remainder is the value .

When given a polynomial function, , we can evaluate at using the Remainder Theorem by

1. Using synthetic division to divide the polynomial by

2. Finding the remainder, which is equal to the value

Example: Use the Remainder Theorem to evaluate the polynomial.

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# Using the Factor Theorem to Solve a Polynomial Equation

The Factor Theorem is another way to analyze polynomial equations. It tells us how the zeros of a polynomial are related to the factors.

According to the **Factor Theorem**, is a zero of if and only if is a factor of .

Given a factor and a third-degree polynomial, we can use the Factor Theorem to factor the polynomial.

1. Use synthetic division to divide the polynomial by .

2. Confirm that the remainder is 0.

3. Write the polynomial as the product of and the quadratic quotient.

4. If possible, factor the quadratic.

5. Write the polynomial as the product of factors.

Essentially, the Factor Theorem is used to completely factor a polynomial into the product of factors, where is the degree of the polynomial. Once a polynomial has been completely factored, it is much easier to determine the zeros of the polynomial.

Example: Show that is a factor of . Find the remaining factors. Use the factors to determine the zeros of the polynomial.

# Using the Rational Zero Theorem to Find Rational Zeros

The Remainder Theorem is used to test whether a rational number is a zero for a given polynomial. But before we can test a number to see if it is a zero, we first need a pool of rational numbers to test.

The **Rational Zero Theorem** states that, if the polynomial

Has integer coefficients, then every rational zero of has the form where is a factor of the constant term and is a factor of the leading coefficient .

Given a polynomial function , we can use the Rational Zero Theorem to find the rational zeros.

1. Determine all factors of the constant term and all factors of the leading coefficient.

2. Determine all possible values of , where is a factor of the constant term and is a factor of the leading coefficient. Be sure to include both positive and negative candidates.

3. Determine which possible zeros are actual zeros by evaluating each case of .

Examples:

1. List all possible rational zeros of .
2. Use the Rational Zero Theorem to find the rational zeros of .

# Finding the Zeros of Polynomial Functions

The Rational Zero Theorem helps us to narrow down the list of possible rational zeros for a polynomial function. Once we have done this, we can use synthetic division repeatedly to determine all of the zeros of a polynomial function.

Given a polynomial function, , we can find zeros by

1. Using the Rational Zero Theorem to list all possible rational zeros of the function.

2. Use synthetic division to evaluate a given possible zero by synthetically dividing the candidate into the polynomial. If the remainder is 0, the candidate is a zero. If the remainder is not zero, discard the candidate.

3. Repeat step two using the quotient found with synthetic division. If possible, continue until the quotient is a quadratic.

4. Find the zeros of the polynomial function.

Example: Find the zeros of .

# Solving Real-World Applications

Now that we’ve seen how to solve polynomial equations, we should be able to take these tools to solve real-world applications.

Examples:

1. A new bakery offers decorated sheet cakes for children’s birthday parties and other special occasions. The bakery wants the volume of a small cake to be 351 cubic inches. The cake is in the shape of a rectangular solid. They want the length of the cake to be four inches longer than the width of the cake and the height of the cake to be one-third of the width. What should the dimensions of the cake pan be?
2. The height of a right circular cylinder is one less than one half the radius. The volume is cubic meters. Find the dimensions of the right circular cylinder.